Extensive and Intensive Margins of Equity Home Bias: Theory and Evidence*

Yuan Liu†
Job Market Paper
University of California, Davis
Oct 15th, 2013

Abstract

A prominent puzzle in international finance is the fact that investors bias their equity portfolio toward domestic assets despite the theoretical gains from diversification. This paper is the first to show that a majority of equity home bias is attributable to the extensive margin, which we define as the proportion of equity holders who do not participate in the foreign equity market. This type of bias calls for a different type of explanation than the intensive margin, which refers to the home biased portfolio conditional on positive holdings of both home and foreign equities. This research helps us understand the nature of overall bias by showing also the portfolio heterogeneity among participants. We incorporate wealth heterogeneity, entry cost, and information acquisition into a rational noisy expectations model to generate both limited participation and portfolio heterogeneity among participants. The model can replicate the share of foreign equities in US equity portfolio in both margins. It makes predictions consistent with the following stylized facts: first, the participation rate is lower in the foreign market than in the home market; second, participation increases in wealth; third, the share of wealth invested in equity increases in wealth; fourth, information acquisition increases in wealth. Finally it also uncovers a new surprising fact: households with foreign equity holdings reduce the degree of international diversification as wealth increases.

JEL Classification: F30; G11; D82

Keywords: Equity Home Bias; Endogenous information acquisition; Wealth heterogeneity; Rational expectations

*I would like to thank Professor Paul Bergin for his continuous advice and encouragement during my Ph.D. studies at UC Davis. Special thanks to Professor Martine Quinzii, Professor Kevin Salyer, and Professor Thanasis Geromichalos for their valuable comments and suggestions. Essential feedbacks were received from participants in the UC Davis Macroeconomics/International Trade brownbag seminar series. I am also grateful to the generous support through UC Davis Humanities Research Award. All errors are my own.

[ecoliu@ucdavis.edu]
1 Introduction

The basic International CAPM model with homogenous investors across the world predicts that a representative investor should always put a fraction of his wealth in equities and the optimal choice is the world portfolio. However, a large number of investors do not hold stocks at all. In addition, the aggregate holdings of domestic stocks is much higher than the share in the world portfolio. The fact that a large fraction of the population in all countries ignore the positive equity premium by not holding any equity is recognized as limited participation puzzle (Mankiw and Zeldes [1991]). The gap between actual holdings of home equities and the share of home equities in the world portfolio is documented as equity home bias (French and Poterba [1991]).

There are few works which link equity home bias to limited participation\footnote{Limited participation is introduced in the context of a closed economy by Guvenen [2009] to explain several asset-pricing puzzles, and in the context of open economy by Kollmann [2010] and Coeurdacier [2011] to explain the consumption real exchange-rate anomaly, or the so called Backus-Smith puzzle. These works all assume exogenously limited participation}. The question studied in the literature of equity home bias is why investors hold more home equities than foreign equities, conditional on non-zero holdings of both. However, before deciding on the share of foreign equities in the portfolio, investors decide on whether to enter the foreign stock market. Actually, the data suggest that the participation rate in the foreign stock market is much lower than in the domestic stock market. We observe on aggregate equity home bias partly because a large number of investors hold only home equities.

This paper is the first one in the literature of equity home bias which emphasizes the significance of limited participation in the foreign equity market. A decomposition of equity home bias developed later in the paper indicates that more than half of the observed home bias in 2005 in the U.S. is actually limited participation in the foreign equity market which we define as the extensive margin bias. Inspired by this observation, we incorporate endogenous information acquisition, wealth heterogeneity and entry costs into a rational noisy expectations model. The model generates endogenously limited participation so that it can distinguish the extensive margin from the intensive margin.

We contribute to the literature also by relating both margins to wealth level and information level. In our story, entry cost discourages poor people away from the stock market. Higher entry cost gives rise to a lower participation rate in the foreign stock market. Investors with foreign equity holdings are in the top of the wealth distribution. Conditional on participation in both the home and foreign markets, demand for foreign equities is lower than for domestic equities since people purchase less foreign information, which costs more.
richer person invests a larger share of wealth in both the home and foreign equities, because he/she has preciser information about equity payoffs. These conclusions drawn from the model are consistent with several stylized facts documented in the literature. Both survey data, Panel Study of Income and Dynamics (Vissing-Jorgensen [2002]) and Survey of Consumer Finance (Nechio [2010]), show that the participation rate in the stock market and the share of wealth invested in stocks increase in wealth, and wealthier participants trade more frequently. Nechio [2010] found that the participation rate is higher in the foreign stock market than in the domestic stock market. Nechio [2010] also found that foreign stock holders are significantly wealthier, have higher non-financial income and acquire more information than domestic stock holders.

According to the model, foreign equity owners are the wealthiest persons in an economy. Their demands for equities is substantially higher than average. How biased these investors’ equity portfolios are may determine the degree of home bias of the country’s equity portfolio. We may expect that diversification increases in wealth since we expect wealthier investors do a better job managing their assets. However, we see in the empirical evidence section that foreign equity owners’ degree of international diversification decreases in wealth. Theoretically, according to our model, international diversification can be monotonic increasing, monotonic decreasing, or first decreasing and then increasing in wealth, depending on the relative increasing speed of the marginal information cost and the relative position of the wealth thresholds. If the marginal information cost increases at the same speed abroad as at home, the person with infinite amount of wealth will perfectly diversify his/her equity portfolio. On the other hand, if the marginal information cost increases much faster abroad than at home, all the rich foreign equity holders are biased towards holding more of home equities than of foreign equities. As wealth increases, they become even more biased towards home. The decreasing diversification in wealth is somehow counter-intuitive and has not yet been documented in the literature to the best knowledge of the author. However, it very conveniently makes the bias less puzzling.

Our work suggests the innegligible importance of portfolio heterogeneity among individual investors. A fully understanding of equity home bias requires proper attentions to both the first-stage extensive margin and the wealthy investors’ equity portfolios. On aggregate, a country’s equity portfolio exhibits home bias because individual investors are holding home biased equity portfolios. First, there is a group of investors who do not diversify at all internationally by holding only home equities. Second, for those who do diversify internationally, the degree of diversification is limited and decreases in wealth. Both the extensive margin
and the decreasing diversification contribute to the high degree of aggregate equity home bias. With reasonable parameter values, the model is able to replicate the share of foreign equity in the equity portfolio of the U.S. at both margins.

We review related literature in section two, describe the model and its predictions in section three, present the decomposition of equity home bias in section four, show the explanatory power of the model through a numerical simulation in section five, document the new stylized fact using data from Survey of Consumer Finance in section six and conclude in section seven.

2 Related Literature

Our work is the first in the literature which proposes a decomposition of equity home bias into the extensive margin bias and the intensive margin bias. The intensive margin bias is the usual equity home bias studied in previous works, while the extensive margin is actually limited participation in the foreign stock market. Therefore, our work connects the literature of equity home bias with the literature of limited participation.

Among many explanations of limited participation in the stock market, perhaps the most powerful one is entry cost\(^2\). A very low entry cost can explain a large proportion of limited participation (Visser-Jorgensen [2002], Haliassos and Michaelides [2003], and Alan [2006]). Entry cost is consistent with cross-country differences in participation rate (Guiso et al. [2002]). A falling entry cost can explain the increasing participation rate (Peress [2005]). A higher entry cost can explain the lower participation in the foreign stock market (Nechio [2010]). Entry cost together with heterogeneity in risk aversion can explain both the low participation in the stock market and the moderate holdings conditional on participation (Gomes and Michaelides [2005]).

As many explanations probably all contribute to equity home bias\(^3\), information asymmetry emerges as a particularly plausible one. Information flows and information cost are important determinants of cross-border equity transactions (Portes et al. [2001] and Portes and Rey [2005]). Information cost is also a major determinant of a country’s weight in US investors’ portfolio (Ahearne et al. [2004]). Even within a country, individual investors (Hu-

\(^2\)In Cao et al. [2005] and Makarov and Schornick [2010a], nonparticipants are more uncertain about stock returns. Gormley et al. [2010] show that lack of insurance against large, negative wealth shock can explain both limited participation and high saving rate. Hong et al. [2004] present a model in which an investor find the market more attractive where more of his peers participate.

\(^3\)Lewis [1999], Karolyi and Stulz [2003], Serce and Vanpee [2007] and Coeurdacier and Rey [2011] provide thorough survey of the literature of equity home bias.
berman [2001] and fund managers (Coval and Moskowitz [1999]) exhibit local bias, which suggests asymmetric information between local and nonlocal investors. Massa and Simonov [2006] argue that familiarity-based investment is not a behavioral bias, but is information driven, because it allows investors to earn higher returns.

Early works propose information asymmetries as explanations of equity home bias rely on exogenous information structures. In Gehrig [1993], Brennan and Cao, [1997], investors are endowed with less precise information about the foreign stock. We contribute to the recent development of endogenous information acquisition in the literature of international finance. Nieuwerburgh and Veldkamp [2009] build a model where a small information advantage in the home equity is amplified by endogenous acquisition of information when investors have limited capacity to process information. Even with initial symmetric information, endogenous information acquisition results in information advantage in the domestic market when the market has frictions represented by costs on holding foreign assets, as in Mondria and Wu [2011], or when the information costs are asymmetric, as in Barron and Ni [2008], Lundtofte [2009], and Eichler [2012]. Different from all these works, we release the hidden assumption that agents are naturally participants in both the home and foreign equity markets. We also link the degree of home bias to the level of wealth by incorporating into our model wealth heterogeneity which is missing in Nieuwerburgh and Veldkamp [2009], Mondria and Wu [2011], Lundtofte [2009] and Eichler [2012].

Wealth heterogeneity is the fundamental differences across households in our model. Its interaction with the asymmetric information cost maps into information heterogeneity through endogenous information acquisition and then into portfolio heterogeneity. Similar structure can be seen in Peress [2004], Peress [2005], Barron and Ni [2008] and Makarov and Schornick [2010b], Peress [2004] and Makarov and Schornick [2010b] try to explain why wealthier households invest a larger fraction of their wealth in risky assets. Peress [2005] relates wealth to the participation decision. In Peress [2004], Makarov and Schornick [2010b] and Peress [2005], equity home bias is not touched since there is not a foreign country in their models. Barron and Ni [2008] interprets the level of wealth as the size of the investment funds. They predict that a smaller fund exhibits a larger degree of home bias. Different from Barron and Ni [2008], we focus also on the first stage participation decision besides the portfolio decision since we do not assume that agents all participate in the stock market.
3 The Model

We develop a rational expectations model in the context of two symmetric countries. The model follows closely the early works on asymmetric information and endogenous information acquisition, especially Hellwig [1980], Verrecchia [1982] and Admati [1985]. The extension which is the key to our desired results lies in three aspects: First, we assume that absolute risk aversion (ARA) in an exponential utility function is decreasing in initial wealth. Second, we have costly endogenous information acquisition. Preciser information about stock payoffs costs more. Third, we have entry costs to justify the limited participation in the home and foreign markets.

The exponential utility function which represents the constant absolute risk aversion (CARA) preference is widely used since it allows for a closed form solution. However, empirical research supports decreasing ARA rather than constant ARA. With CARA preferences, two agents who have the same level of ARA demand the same dollar amount of risky assets regardless of their wealth. With decreasing ARA preferences, the richer one who exhibits a lower level of ARA demands a larger dollar amount of risky assets. Constant relative risk aversion (CRRA) preference is a special case of decreasing ARA preference. It is a better choice than CARA preference when a closed form solution is not essential. Let’s consider a CRRA preference with RRA being \( a \). ARA of this preference is \( \frac{a}{W} \), which is similar to the ARA of the preference used in our model \( \left( \frac{a}{W_0} \right) \). The difference between CRRA preference and the preference used in our model is as follows: In CRRA preference, ARA is decreasing in current wealth, thus it may be changing over time as current wealth changes. In the preference used in our model, ARA is decreasing in initial wealth, thus it is constant over time. In a static model like ours, the difference does not matter. The utility function in our model is a local approximation of CRRA preference. Agents with CRRA preferences will spend a constant share of wealth in risky assets. If agents are not allowed to purchase information in our model, the share of wealth invested in equities is the same across agents with different amount of initial wealth, as predicted by CRRA preference. The same utility function as ours can be seen in Peress [2004], Peress [2005], Barron and Ni [2008] and Makarov and Schornick [2010b].

The information cost function either at home or abroad is increasing and convex. We think of the information cost as money spent on services from a professional and/or time spent on collecting and analyzing information by self. The more commission fees that an investor pays to his fund manager, and/or the more time and effort that an investor spends on collecting and analyzing information, the preciser his predictions are, thus the more
likely the portfolio returns higher and less risky. Not only the information cost but also the marginal information cost is increasing. It becomes harder and harder to collect an extra piece of information. It may take not much time and money for an investor to reach the average investment performance. However, an investor should spend a lot of time, effort and money on information if he wants to reach the performance of Warren Buffet.

Information cost asymmetry lies both in the level and on the margin. We assume that the information cost as well as the marginal information cost are higher abroad than at home. Not only collecting the same amount of information, but also acquiring the same amount of extra information at any information level cost more abroad than at home. Due to this asymmetry, agents always purchase more information at home than abroad.

If agents in our model are not allowed to purchase information, they all spend the same proportion of their wealth on equities. In other words, the model in the absence of information acquisition predicts that wealthier ones hold a larger dollar amount of equities. When agents are allowed to buy information about equity payoffs before the purchase of any equity, wealthier ones buy more information because they will demand a larger dollar amount of equities. Wealth heterogeneity maps into information heterogeneity through endogenous information acquisition, and then into portfolio heterogeneity. Wealthy ones who have preciser information end up spending a larger share of their wealth on equities. The share of wealth in equities is no longer the same across agents but increases in initial wealth. In the absence of entry cost, all agents, regardless of their wealth levels, are naturally participants in the equity market. However, a poor agent will purchase a small dollar amount of equities since he has very noisy information about the payoffs. When the investors are imposed an entry cost, benefits of holding such a small number of risky equities, to the poors, is too low to justify the entry cost. Entry cost asymmetry working together with wealth heterogeneity gives rise to the wealth threshold of participation in the equity market. Only those with initial wealth above the threshold choose to participate. Participants are wealthy ones who have precise information and purchase a large number of equities once they go to the market. Their benefits from holding such a large number of equities is high enough to justify the entry cost. Higher entry cost raises the wealth threshold and thus discourages more persons away from the equity market.

We can split equity home bias into the extensive margin bias and the intensive margin bias once we have a model which generates endogenously limited participation in the equity market. Higher entry cost in the foreign equity market results in a higher wealth threshold of participation in the foreign market. Agents with initial wealth above the threshold of
participation in the domestic market but below the threshold of participation in the foreign market buy only the home equity. Their equity portfolios are completed biased towards home, and form the extensive margin home bias. Agents who are rich enough to pay the entry costs to participate in both markets purchase more information at home, thus put a higher share of wealth in the home equity. Lower demands for foreign equity than for home equity conditional on positive holdings of both is named the intensive margin bias. Both the lower participation rate in the foreign stock market and the lower demands for foreign equities conditional on participation contribute to the aggregate home bias. Two margins together give more power to the model towards explaining the high degree of equity home bias.

Agents in the top wealth group demand for a very large number of both home equities and foreign equities. They have disporoportionate influence on asset demand in the market. With the marginal information cost increasing at the same speed at home and abroad, the model predicts that the person with infinite amount of wealth will diversify perfectly internationally. On the other hand, if the marginal information cost increases much faster abroad than at home, even the person with infinite amount of wealth hold an equity portfolio that is severely biased towards home. Intuitively, we may expect that diversification increases in wealth since we think that wealthy ones do a better job than average managing their wealth. However, empirical evidence suggests a negative correlation between international diversification and wealth. Although the share of wealth in both home equities and foreign equities increase as wealth increases, the former increases at a faster speed. This decreasing diversification is conveniently consistent with the observed high degree of equity home bias. Although there is a group of investors who hold only the home equity, their demands for equities is much less than the demands from the top wealth group who put part of their investment across borders. If these guys’ equity portfolios are not home biased, it is hard to explain the high degree of aggregate home bias. Fortunately or Unfortunately, these very rich persons on average are biased towards home, and are even more biased with a larger amount of wealth.

Based on our understandings of the model and the empirical evidences. We think that a country exhibits a high degree of equity home bias because individual investors on average hold home biased equity portfolios. There is a group of investors who do not diversify at all internationally. For those who do diversify, diversification is limited and it decreases in wealth.

---

4In 2007, the top 1% households in the wealth distribution owned 38.3% of all the directly or indirectly held stocks in the U.S. (Wolff [2010]).
### 3.1 Economic Setting

There are two symmetric countries: home and foreign. Each country has infinitely many agents measured one. Each agent is endowed with some amount of initial wealth. Initial wealth follows the same Pareto Distribution in two countries: the lower bound is $W$ ($W > 0$) and the shape Parameter is $\lambda$ ($\lambda > 1$). Each country issues one risky equity. The home equity and the foreign equity are independent from and identical to each other in terms of the payoff structure and the supply.

The agents make three consecutive decisions in four periods: at time one, the agents observe their initial wealth and decide whether to enter the home and/or the foreign equity market; at time two, the agents decide the amount of information to purchase in each market which they choose to participate in at the previous period; at time three, the agents observe the private information which they purchased at the previous period as well as the price of each equity, then they make a portfolio decision; at the last period, equity payoffs are revealed, and the agents consume.

Everything is measured in one single consumption goods. The risk free rate of return is $r$. The subscript $i$ denotes the issuer of an equity. For $i = h, f$, equity payoff is $\tilde{R}_i$, per capita supply of each equity is $\tilde{Z}_i$. Agent $k$’s private signal about the return on equity $i$ is $\tilde{Y}_{ik} = \tilde{R}_i + \tilde{\epsilon}_{ik}$. We assume that $\tilde{R}_i$, $\tilde{Z}_i$ and $\tilde{\epsilon}_{ik}$ follow a joint normal distribution for $i = h, f$ and all $k$.

$$
\begin{bmatrix}
\tilde{R}_i \\
\tilde{Z}_i \\
\tilde{\epsilon}_i
\end{bmatrix}
\sim N
\begin{bmatrix}
\tilde{R} \\
\tilde{Z} \\
0
\end{bmatrix}
\begin{bmatrix}
\frac{1}{V} & 0 & 0 \\
0 & U & 0 \\
0 & 0 & \frac{1}{S_i}
\end{bmatrix}
$$

$S_i$ is the precision of the private information in market $i$. A high $S_i$ means that the private signal $Y_i$ is close to the true payoff $R_i$ with a high probability. Agents purchase $S_i$. A bigger $S_i$ costs more. All agents have same information cost function. The information cost function in the market $i$ is $c_i(S_i)$. We assume that $c_i(S_i)$ is convex and the information in the foreign market is more costly:

- For $i = h, f$: $c_i(0) = 0$
- For $i = h, f$ and all $S \geq 0$: $c'_i(S) > 0$, $c''_i(S) \geq 0$ and $c_h(S) < c_f(S)$, $c'_h(S) < c'_f(S)$

\footnote{We put a hat on the top to indicate that it is a random variable. When the hat is removed, it is the realization of the random variable.}
\footnote{With no harm to our analysis, we suppress the subscript $k$.}
Each agent’s preference is represented by an exponential function:

$$u(W) = - \exp\left(- \frac{W}{\rho}\right)$$  \hspace{1cm} (1)

$$W = W_0r + \sum_{i=h,f} \{D_i(R_i - rP_i) - c_i(S_i) - F_i\}$$  \hspace{1cm} (2)

$W$ and $W_0$ are final and initial wealth. $\rho^{-1}$ is the absolute risk aversion. We call $\rho$ the risk tolerance. $F_i$ is the entry cost in market $i$. We assume that the entry cost is higher in the foreign market $F_f > F_h$. $D_i$ is the demand for equity $i$. $R_i$ and $P_i$ are the revealed equity payoff and the price.

Following the literature\(^7\), we assume that absolute risk aversion (ARA) decreases in initial wealth

$$ARA = \frac{1}{\rho} = \frac{a}{W_0}$$  \hspace{1cm} (3)

If we have final wealth $W$ instead of initial wealth $W_0$ in equation (3), the preference is a constant relative risk aversion (CRRA) preference.

$$RRA = W \times ARA = a$$  \hspace{1cm} (4)

The exponential utility function together with this assumption about ARA makes effectively our preference just a CRRA preference since we have a static not a dynamic model. A utility function which directly exhibits CRRA does not allow a closed form solution.

### 3.2 The Solution

Each agent maximizes the expected utility. At period three, an agent observe his private signal about the equity payoffs and the equity prices. Therefore the expectation is based on the conditional distribution of equity payoff on the revealed private signal and the price. Therefore an agent maximizes $E_{R|Y_i, P_i} (u(\cdot))$ by choosing the optimal portfolio at period three. The highest expected utility achieved with the optimal portfolio, and denoted by $v$, depends on the revealed private signal $Y_i$ and price $P_i$. At period two, the private signal and the price are random variables following a joint normal distribution, based on which the expected utility at period two is calculated. At period two, agents maximize $E_{Y_i, P_i} (v)$ by purchasing the optimal amount of private information. We can represent agents’ expected

\(^{7}\)Barron and Ni [2008], Peress [2004], Peress [2005], Makarov and Schornick [2010b]
utility in a nested form as follows:

$$\max \mathbb{E}_{\hat{R}, \hat{Y}, \hat{P}} (u(.)) = \max \mathbb{E}_{\hat{Y}, \hat{P}} \left( \mathbb{E}_{\hat{R} | \hat{Y}, \hat{P}} (u(.)) \right)$$  \hspace{1cm} (5)$$

At period one, each agent makes the participation decision by comparing the expected utility of not participating in the equity market with the expected utility achieved by the optimal portfolio purchasing and the optimal information acquiring. We present the exact solution to the model backward.

1 Portfolio Decision

An agent’s initial belief about equity payoff is: $\hat{R}_i \sim N(\bar{F}, V^{-1})$ for $i = h, f$. Once the private signal and the price are revealed, an agent updates his belief to the posterior one: $\hat{R}_i \sim N(E_i, V^{-1}_i)$ for $i = h, f$. The posterior mean $E_i$ and precision $V_i$ depend on the revealed private signal and the price. Based on the posterior distribution of equity payoff, an agent derives the optimal demand $D_i$ for equity $i$ which is as follows:

$$D_i = \rho V_i (E_i - r P_i)$$  \hspace{1cm} (6)$$

The demand for risky equity is high if the risk tolerance $\rho$ is high, information about the equity payoff $V_i$ is very precise, or the expected excess return $E_i - r P_i$ is high. $\rho$ is linear in initial wealth. If all agents have the same posterior belief about the equity payoff, they spend the same share of initial wealth in equity. It also means that a richer one has a higher absolute amount of equity. We will see later that a richer one also has preciser information about the equity payoff, so he/she actually spends a larger share of wealth in equity.

$P_i$ is the market clearing price which equates the aggregate demand to the aggregate supply. Agents’ demands affect, and on the other hand are determined by, the market clearing price. Demand and price are simultaneously and endogenously determined in the model. [Hellwig 1980] and [Admati 1985] proved the existence of a rational expectations equilibrium price in a single asset model and a multi-asset model respectively. We restate their results as follows:

**Proposition 1.** There exists a unique rational expectations equilibrium price for $i = h, f$:

$$\hat{P}_i = A_0 + A_1 \hat{R}_i - A_2 \hat{Z}_i$$  \hspace{1cm} (7)$$

$^8$See appendix A for the derivation of the optimal portfolio.
And:

\[
\begin{align*}
A_0 &= \frac{\bar{\rho}}{r}(\bar{\rho}V + \bar{\rho}\frac{Q^2}{U} + Q)^{-1}(V\bar{R} + \frac{Q}{U}\bar{Z}) \\
A_1 &= \frac{1}{r}(\bar{\rho}V + \bar{\rho}\frac{Q^2}{U} + Q)^{-1}(Q + \bar{\rho}\frac{Q^2}{U}) \\
A_2 &= \frac{1}{r}(\bar{\rho}V + \bar{\rho}\frac{Q^2}{U} + Q)^{-1}(I + \bar{\rho}\frac{Q^2}{U})
\end{align*}
\]

\(\bar{\rho} = \int \rho_k dk\) is the average risk tolerance among all market participants, \(Q = \int \rho_k S_k dk\) is the weighted average precision of all participants’ private information.

**Corollary 1.1.** For \(i = h, f\), the equity payoff \(\tilde{R}_i\), the private signal \(\tilde{Y}_i\), and the price \(\tilde{P}_i\) follow a joint normal distribution:

\[
\begin{bmatrix}
\tilde{R}_i \\
\tilde{Y}_i \\
\tilde{P}_i
\end{bmatrix}
\sim
N
\begin{bmatrix}
R & V^{-1} & A_1V^{-1} \\
\bar{R} & V^{-1} & V^{-1} + S_i^{-1} & A_1V^{-1} \\
A_0 + A_1\bar{R} - A_2\bar{Z} & A_1V^{-1} & A_1V^{-1} & A_1V^{-1}
\end{bmatrix}
\]

(8)

Therefore, in the posterior distribution \(\tilde{R}_i|Y_i, P_i \sim N(E_i, V_i^{-1})\):

\[
E_i = \bar{R} + B_{i1}(Y_i - \bar{R}) + B_{i2}(P_i - (A_0 + A_1\bar{R} - A_2\bar{Z}))
\]

(9)

\[
V_i^{-1} = V^{-1} - B_{i1}V^{-1} - B_{i2}A_1V^{-1}
\]

(10)

Where

\[
\begin{bmatrix}
B_{i1} & B_{i2}
\end{bmatrix}
\begin{bmatrix}
V^{-1} + S_i^{-1} & A_1V^{-1} \\
A_1V^{-1} & A_1^2V^{-1} + A_2^2U
\end{bmatrix}
= \begin{bmatrix}
V^{-1} & A_1V^{-1}
\end{bmatrix}
\]

There are infinitely many agents in the market, each of them possesses a piece of noisy but unbiased private information. Market clearing price aggregates all private information which is delivered by agents’ demands. The price conveys the public information, in which all the noises contained in each piece of private information disappear, since there are infinitely many pieces of private information and all of them are unbiased.

Price is linear in the true payoff and the supply. Agents can perfectly infer the equity payoff from the price and and the private signal if the supply is certain. Therefore, a stochastic supply is essential to the functioning of the model.
2 Information Acquisition Decision

At period three, the highest expected utility achieved with the optimal portfolio is

\[ v = -\exp\left(\frac{-W_0 r}{\rho}\right) \prod_{i=h,f} \exp\left(\frac{F_i + c_i(S_i)}{\rho} - \frac{(E_i - rP_i)^2 V_i}{2}\right) \] (11)

\(E_i\) and \(V_i\) are functions of the revealed private signal \(Y_i\) and the price \(P_i\). Therefore \(v\) depends on \(Y_i\) and \(P_i\). At period two, \(Y_i\) and \(P_i\) are random variables following a joint normal distribution. The variance-covariance structure of this distribution is determined by the precision of the private signal. Agents choose a distribution over which the expectation of \(v\) is highest by choosing the precision of the private signal \(S_i\).

**Proposition 2.** \(^9\) The optimal precision level of private information about equity \(i\) is \(S^*_i = \max\{0, \tilde{S}_i\}\) and \(\tilde{S}_i\) is implicitly given by:

\[ c'_i(\tilde{S}_i) = \frac{W_0 1}{2a V_i} \] (12)

Where

\[ V_i = V + \tilde{S}_i + \frac{Q^2}{U} \] (13)

The optimal precision of private information equates the marginal cost to the marginal benefit of information acquisition. The marginal cost is the marginal information cost. The marginal benefit is decreasing in \(V_i\) and increasing in \(W_0\). We can see from equation (13) that the posterior precision of an agent’s belief about the equity payoff is the sum of the prior precision \(V\), the precision of the private signal \(S_i\) and the precision of the public signal \(\frac{Q^2}{U}\). In other words, the final set of information possessed by each agent includes the common prior information, the purchased private information and the public information conveyed by the price. The benefit from getting an extra piece of private information, to an agent who already has very precise information (a high \(V_i\)), is small. This decreasing margin, however, is scaled by the demand for equity. More equities that an agent purchases, more benefits that the agent receives from purchasing an extra piece of information. Since the demand for equity is increasing and linear in initial wealth (see equation (6)), the marginal benefit is increasing and linear in initial wealth. The scaling effect dominates so that a richer one purchases more

---

\(^9\) See Appendix B for the derivation of \(v\).

\(^{10}\) See Appendix B for the proof.
information. Intuitively speaking, a wealthier agent purchases more information because he who will hold a lot equities benefits more from a richer set of information. This conclusion is stated in Corollary 2.1.

**Corollary 2.1.** The optimal amount of information acquisition is a nondecreasing function of an agent’s initial wealth.

\[
\frac{\partial S_*^i}{\partial W_0} = \begin{cases} 
0 & S_*^i = 0 \\
\frac{c'_i}{2a(c'_i)^2 + W_0 c''_i} > 0 & S_*^i > 0
\end{cases}
\]

The amount of information acquisition can not be negative (we do not allow agents sell information). The optimal information acquisition either increases in wealth and is positive, or it stays at zero. Only those agents with initial wealth above a certain threshold purchases a positive amount of information.

**Corollary 2.2.** There exists a wealth threshold \( \bar{W}_i^s \) which is given by:

\[
\bar{W}_i^s = 2a(V + \frac{Q^2}{U})c'_i(0)
\]

Agents with initial wealth below the threshold will not purchase information.

*Proof.\* \( \tilde{S}_i(W) < 0 \) if \( W < \bar{W}_i^s \) because \( \frac{\partial \tilde{S}_i}{\partial W_0} > 0 \) and \( \tilde{S}_i(\bar{W}_i^s) = 0 \). Since we do not allow negative information acquisition, \( S^*(W) = \max\{0, \tilde{S}_i(W)\} = 0 \) if \( W < \bar{W}_i^s \) \( \square \)

**Corollary 2.3.** Agents who acquire positive amount of information purchase more information about the home equity than about the foreign equity: \( S^*_h \geq S^*_f \). The equality holds if and only if \( S^*_h = S^*_f = 0 \)

*Proof.\* If \( \tilde{S}_h > \tilde{S}_f \) then \( S^*_h \geq S^*_f \).

Suppose \( \tilde{S}_h \leq \tilde{S}_f \), then \( c'_h(\tilde{S}_h) \leq c'_f(\tilde{S}_f) \).

Then

\[
2a(V + \tilde{S}_h + \frac{Q^2}{U}) \frac{c'_h(\tilde{S}_h)}{W_0} < 2a(V + \tilde{S}_f + \frac{Q^2}{U}) \frac{c'_f(\tilde{S}_f)}{W_0}
\]

This contradicts Proposition 2. Thus \( \tilde{S}_h > \tilde{S}_f \) and \( S^*_h \geq S^*_f \). \( \square \)

For those agents who indeed purchase some information, they purchase more at home than abroad since the information at home cost less.
Corollary 2.4. The cut-off level of wealth for the foreign information acquisition is higher than for the home information acquisition.

\[ \bar{W}_f^* > \bar{W}_h^* \]

Proof. It follows from the fact that \( c'_f(0) > c'_h(0) \) and Corollary 2.2.

We now turn to the effect of wealth on an agent’s investment in equity. Empirical evidence suggests that richer investors tend to put larger shares of their wealth in equity. Our model can generate a conclusion which is consistent with this observation. \( \sigma_i \) is the share of wealth invested into risky equity \( i \), and it is defined as follows:

\[ \sigma_i = \frac{D_i P}{W_0} \] (14)

Corollary 2.5. If the expected equity holding is positive, the expected share of initial wealth in equity increases in initial wealth.

Proof. The expected value of investment in equity \( i \) is:

\[
\mathbb{E}_{\tilde{Y}_i, \tilde{P}_i}(D_i P_i) = \rho (V + S_i + \frac{Q^2}{U}) C
\]

\[
C = \frac{\bar{Z} F(\rho V + \bar{\rho} \frac{Q^2}{U} + Q) - \bar{Z}^2 - \bar{\rho}Q - U}{r(\bar{\rho} V + \bar{\rho} \frac{Q^2}{U} + Q)^2}
\]

The expected share of wealth invested in equity \( i \) is:

\[
\mathbb{E}(\sigma_i) = \mathbb{E}_{\tilde{Y}_i, \tilde{P}_i}(\frac{D_i P}{W_0}) = \frac{1}{a} (V + S_i + \frac{Q^2}{U}) C
\]

The effect of wealth on the expected share of wealth invested in equity \( i \) is:

\[
\frac{\partial \sigma_i}{\partial W_0} = \frac{C}{a} \frac{\partial S_i^*}{\partial W_0}
\]

Since \( \frac{\partial S_i^*}{\partial W_0} \) is non-negative, \( \mathbb{E}(D_i P) \) and \( \frac{\partial \sigma_i}{\partial W_0} \) have the same sign. \( \square \)

\(^{11}\)See Appendix C for the calculation of mean investment value.

\(^{12}\)See Appendix C for a discussion of a negative expected value of investment in equity.
If all the agents share the same set of information about equity pay-offs, they would invest the same share of their initial wealth into equities regardless of their absolute levels of initial wealth. However, the amount of information possessed by each agent differs with wealth. Richer ones possess more information about because they spent more to acquire more private information. With a more accurate prediction about future equity payoffs, a wealthier agent invests a larger share of his initial wealth in equity.

3 Participation Decision

At period one, agents decide whether to participate in the equity market by comparing the expected utility of participation with the expected utility of non-participation.

Proposition 3. Agents participate in market \( i \) \( (i = h, f) \) if the benefit of doing so exceeds the cost:

\[
F_i + c_i(S^*_i) < \frac{W_0}{2a} \left( \ln \left( \frac{V + \frac{Q^2}{U} + S^*_i}{B} \right) + A \right)
\]  

(16)

Proof. Agents’ expected utility in the second period evaluated at the optimal level of information acquisition, which is also the expected utility of participation is:

\[
v^p = -\exp\left(\frac{-W_0r}{\rho}\right) \prod_{i=h,f} \exp\left(\frac{F_i + c_i(S^*_i)}{\rho}\right) \exp\left(\frac{-A}{2}\right) \sqrt{\frac{B}{(V + S^*_i + \frac{Q^2}{U})}}
\]

(17)

Purchasing equity \( i \) adds to the expected utility:

\[
\exp\left(\frac{F_i + c_i(S^*_i)}{\rho}\right) \exp\left(\frac{-A}{2}\right) \sqrt{\frac{B}{(V + S^*_i + \frac{Q^2}{U})}}
\]

It increases the expected utility if and only if:

\[
\exp\left(\frac{F_i + c_i(S^*_i)}{\rho}\right) \exp\left(\frac{-A}{2}\right) \sqrt{\frac{B}{(V + S^*_i + \frac{Q^2}{U})}} < 1
\]

(18)

This inequality condition is equivalent to:

\[
F_i + c_i(S^*_i) < \frac{W_0}{2a} \left( \ln \left( \frac{V + \frac{Q^2}{U} + S^*_i}{B} \right) + A \right)
\]
Corollary 3.1. The benefit of participation in market $i$ ($i = h, f$) increases in initial wealth.

Proof. The benefit of participation in market $i$ is:

$$B_i(W_0) = \frac{W_0}{2a} \left( \ln \left( \frac{V + \frac{Q^2}{U} + S_i^*}{B} \right) + A \right)$$

The effect of wealth on the benefit is:

$$B'_i(W_0) = \frac{1}{2a} \left( \ln \left( \frac{V + \frac{Q^2}{U} + S_i^*}{B} \right) + A \right) + \frac{W_0}{2a(V + \frac{Q^2}{U} + S_i^*)} \frac{\partial S_i^*}{\partial W_0}$$

And

$$\frac{V + \frac{Q^2}{U} + S_i^*}{B}$$

$$= (V + \frac{Q^2}{U} + S_i^*) \left( 1 - A_1r \right)^2 \frac{1}{V} + \left( A_1r \right)^2 \frac{U}{Q^2}$$

$$= (V + \frac{Q^2}{U}) \left( (1 - A_1r)^2 \frac{1}{V} + (A_1r)^2 \frac{U}{Q^2} \right) + S_i^* \left( (1 - A_1r)^2 \frac{1}{V} + (A_1r)^2 \frac{U}{Q^2} \right)$$

$$= 1 + \frac{Q^2}{U} \frac{1}{V} \left( 1 - A_1r - \frac{A_1r}{\frac{Q^2}{U} V} \right)^2 + S_i^* \left( (1 - A_1r)^2 \frac{1}{V} + (A_1r)^2 \frac{U}{Q^2} \right)$$

$$> 1$$

Therefore

$$B'_i(W_0) > 0$$

A richer person benefits more from investing in the risky equities because he has preciser information about the payoffs and will hold a larger amount of equities once he goes to the market. However, the cost of participation is also increasing in initial wealth since a richer person buys more information and therefore pays a higher cost. It turns out that benefits increases faster than costs. Therefore participants in the equity market have initial wealth passing a certain threshold.
Corollary 3.2. There exists a wealth threshold $\bar{W}_i$ which is given by:

$$F_i + c_i \left( S_i^* \left( \bar{W}_i \right) \right) = B_i(\bar{W}_i)$$

Only agents with initial wealth above the threshold $\bar{W}_i$ participate in market $i$.

Proof. Equation (16) can be written as:

$$F_i < B_i(W_0) - c_i \left( S_i^* \left( W_0 \right) \right)$$  \hspace{1cm} (19)

Define function $G_i(.)$ as:

$$G_i(W_0) = B_i(W_0) - c_i \left( S_i^* \left( W_0 \right) \right)$$  \hspace{1cm} (20)

Thus $G_i(\bar{W}_i) = F_i$. The first order derivative is:

$$G_i'(W_0) = B_i'(W_0) - c_i'(S_i^*) \frac{\partial S_i^*}{\partial W_0}$$

$$= \frac{1}{2a} \left( \ln \left( \frac{V + Q^2}{U} + S_i^* \right) + A \right)$$

$$> 0$$

$W_0 > \bar{W}_i$ implies that $G_i(W_0) > G_i(\bar{W}_i) = F_i$, the benefit of participation exceeds the cost. An agent participates in country $i$ market only if his initial wealth $W_0$ is higher than the cut-off level $\bar{W}_i$. \hfill $\square$

Corollary 3.3. The cut-off wealth for participation in market $i$ increases in the entry cost in market $i$.

$$\frac{\partial \bar{W}_i}{\partial F_i} > 0$$

Proof.

$$\frac{\partial \bar{W}_i}{\partial F_i} = \frac{1}{G_i''(\bar{W}_i)} > 0$$  \hspace{1cm} (21)

$\square$

Corollary 3.4. The Wealth threshold in the home equity market is lower than in the foreign equity market.

$$\bar{W}_h < \bar{W}_f$$
Proof.

\[ G'_i(W_0) = \frac{1}{2a} \left( \ln \left( \frac{V + \frac{Q^2}{U} + S_i^*}{B} \right) + A \right) \]

According to Corollary 2.3, \( S^*_h(W_0) \geq S^*_f(W_0) \). \( G'_h(W_0) \geq G'_f(W_0) > 0 \)

Since \( G_h(0) = G_f(0) = 0 \), \( G_h(W_0) \geq G_f(W_0) \).

Suppose \( \bar{W}_h > \bar{W}_f \), then we have

\[ G_h(W_h) > G_h(W_f) \geq G_f(W_f) \]

Since \( F_h = G_h(\bar{W}_h) \) and \( F_f = G_f(\bar{W}_f) \), it implies that \( F_h > F_f \), which contradicts the assumption that \( F_h < F_f \).

Only to those who are rich enough, benefits of investing in equity is high enough to justify the entry cost. In other words, the fixed entry cost discourages poor ones away from the equity market. A higher fixed cost discourages away more agents.

### 3.3 Equity Home Bias at Individual Level

The cut-off levels of wealth for participation in the home and foreign market divide the whole population into three groups: the top wealth group invests in both the home and foreign equity markets; the bottom wealth group does not invest in equity; the group in between invests in only the home market. Individuals belonging to the group in between hold completely home-biased equity portfolios. Their contributions to the aggregate level of equity home bias is recognized as the extensive margin bias. Individuals in the top wealth group invest in both the foreign and the home market, however, they put a larger share of their wealth in the home market than in the foreign market. The home-biased portfolios conditional on positive holdings of foreign equities form the intensive margin bias.

The model successfully generates heterogenous equity portfolios among individual investors. We would like to measure the level of diversification achieved by each agent’s equity portfolio, so we know how one portfolio differs from another. We use the share of foreign equities in an agent’s equity portfolio as the measure of diversification. An agent’s degree of
international diversification $d$ is defined as follows:

$$d = \frac{\mathbb{E}(D_f P_f)}{\mathbb{E}(D_h P_h + D_f P_f)} = \frac{V + \frac{Q^2}{U} + S_f^*}{V + \frac{Q^2}{U} + S_h^* + V + \frac{Q^2}{U} + S_f^*}$$  \hspace{1cm} (22)

Agents with initial wealth above the domestic market participation wealth threshold but below the foreign market participation wealth threshold hold equity portfolios that consist of only domestic equities. The degree of diversification of these portfolios is zero. A fully diversified portfolio, or a portfolio which does not exhibit home bias, has a degree of diversification being 0.5. The top wealth group do diversify international. They all hold some foreign equities. The questions are how diversified these rich persons are, and how does wealth affect the degree of diversification?

$$d = 0 \text{ if } \bar{W}_h \leq W_0 \leq \bar{W}_f$$

$$0 < d \leq 0.5 \text{ if } W_0 > \bar{W}_f$$

The answer depends on the relative increasing speed of the marginal information cost as well as the relative position between the participation wealth threshold and the information acquisition wealth threshold in the foreign market.

**Proposition 4.** The degree of international diversification of an individual investor’ equity portfolio converges to $\bar{d} = 1 - (1 + \sqrt{c''_h c''_f})^{-1}$ as initial wealth goes to infinity.

**Proof.** Define $b$ as follows:

$$b = \frac{V + \frac{Q^2}{U} + S_f^*}{V + \frac{Q^2}{U} + S_h^* + V + \frac{Q^2}{U} + S_f^*} = \frac{c_h(S_h^*)}{c_f(S_f^*)}$$

The effect of wealth on $b$ is:

$$\frac{\partial b}{\partial W_0} = \frac{c_h c_f \partial S_h^*}{c_f W_0} - \frac{c_h c_f \partial S_f^*}{c_f W_0}$$

Therefore

$$\frac{\partial b}{\partial W_0} > 0 \text{ if } b < \sqrt{\frac{c''_h}{c''_f}}$$

$$\frac{\partial b}{\partial W_0} < 0 \text{ if } b > \sqrt{\frac{c''_h}{c''_f}}$$

Since $d = 1 - (1 + b)^{-1}$
Finally we have
\[
\frac{\partial d}{\partial W_0} > 0 \text{ if } d < 1 - (1 + \sqrt{c_h'}} (c_f'})^{-1}
\]
\[
\frac{\partial d}{\partial W_0} < 0 \text{ if } d > 1 - (1 + \sqrt{c_h'}} (c_f'})^{-1}
\]

If \( c_h'' = c_f'' \), \( \bar{d} = 0.5 \). The equity portfolio held by an individual investor converges to a perfectly diversified one as initial wealth approaches infinity. In other words, Warren Buffet should be very close to be fully diversifying internationally. On the other hand, if \( c_h'' \) is much smaller than \( c_f'' \), even the richest person in the world holds an equity portfolio that is severely biased toward the home market.

The second order derivative of the information cost function measures how fast the marginal information cost increases. If the information cost function is quadratic as the one used in our numerical simulation, the second order derivative is the coefficient on the quadratic term. \( c_h'' = c_f'' \) means that the home and foreign information cost functions have the same quadratic term. In other words, the marginal information cost increases at the same speed at home and abroad. As wealth increases, information acquisition increases. Eventually, the quadratic term dominates the linear term in the information cost function. Home information and foreign information will be equally costly to the richest person who acquires substantial amount of information. Therefore the person with infinite amount of wealth will perfectly diversify his equity portfolio internationally. On the other hand, if the marginal information cost increases much faster abroad than at home (the foreign information cost function has a much bigger quadratic term), as the quadratic term grows, it becomes relatively more and more expensive buying foreign information than buying home information. Although the information purchased in both markets increase in wealth, it increases much slower abroad than at home. Information gap increases, and asset demand gap increases. International diversification decreases in wealth.

The participation wealth threshold is \( \bar{W}_h \) in the home market and \( \bar{W}_f \) in the foreign market. The information acquisition wealth threshold is \( \bar{W}_h^* \) in the home market and \( \bar{W}_f^* \) in the foreign market. Since the wealth threshold, either of participation or of information acquisition, is higher in the foreign market than in the domestic market, there are in total six different ranks of the four thresholds. Let’s look at the three ranks\(^{13}\) pictured in Figure

\(^{13}\)In all three cases: \( W_h < W_h^* \). Varying the relative position between \( \bar{W}_h \) and \( \bar{W}_h^* \) does not have an impact on the relationship between wealth and diversification.
to discuss how it can affect the relationship between diversification and wealth:

Figure 1: Diversification, wealth, and Ranks of Wealth thresholds

If $W^s_h$ and $W^s_f$ are both higher than $W_f$ (case (1) in Figure 1), we can re-divide the top wealth group (with initial wealth above $W_f$) into three subgroups. First, those with initial wealth higher than $W_f$ but lower than $W^s_h$ invest in both the home and foreign equities but do not buy any information. Home equity and foreign equity are identical to them, so they fully diversify internationally. In other words, they split their equity investment equally between the home and foreign market. Second, those with initial wealth above $W^s_h$ but below $W^s_f$ buy home information but not foreign information. They know more about the home equity than about the foreign equity. Therefore, they invest more in the home market. As initial wealth goes up, an investor purchases more information. All the information is about the home equity. The equity portfolio becomes more biased towards the home market. The degree of diversification decreases in initial wealth. Third, those with initial wealth above $W^s_f$ purchase information about both the home and foreign equities. Diversification converges to $\bar{d}$ as wealth goes to infinity. Depending on how diversified the person who has initial wealth equal to $W^s_f$ is, diversification can be increasing or decreasing in wealth. If the agent, who has initial wealth $W^s_f$ and start buying a positive amount of foreign information, is less diversified than $\bar{d}$, diversification increases as wealth increases. On the other hand, if the agent with just enough initial wealth to buy a little bit foreign information is more diversified than $\bar{d}$, diversification decreases as wealth increases.

If only $W^s_f$ is above $W_f$ (case (2) in Figure 1), the first subgroup disappears. If $W^s_h$ and $W^s_f$ are both lower than $W_f$ (case (3) in Figure 1), the first and second subgroups disappear.

Among those who are wealthy enough to invest in both the home and foreign market,
international diversification of their equity portfolio can be monotonically increasing, monotonically decreasing or first decreasing then increasing (non-monotonic) in initial wealth. We summarize all the possible situations in Figure 1. However, there is always only one truth. That truth is, according to the empirical evidences documented later in this paper, that international diversification monotonically decreases in wealth. The model can generate this negative correlation between diversification and wealth with the assumption that the marginal information cost increases faster abroad than at home.

In the model, investors buy information to reduce the variance, which represents the level of risk, of the return to their investment. A less risker portfolio gains the investor more utility. To match the decreasing diversification, we assume that the information cost is higher at level, at margin, and at the margin of the margin abroad than at home. Due to information cost asymmetry, investors increase their holdings of foreign equities much slower than of the domestic equities as wealth increases. Asymmetric information cost determines the asymmetric equity demand.

In the reality, a foreign country differs from the home country in many aspects: the culture, the language, natural resource endowments, the history, the political system, accounting rules, the regulatory system, consumption preferences, and so on. All these differences potentially could contribute to the information cost asymmetry in the equity market. It is naturally easier for investors to observe things going on in the home economy than what happened across borders. Obtaining knowledge of the status of a foreign economy involves intentionally searching and researching news and publications related to the foreign country. While information about the well-being of the home economy can be obtained effortlessly when you listen to NPR on the way to work, and when you read newspapers or watch TV. These information about the basics of an economy serves reducing the variance of the equity return, investors with these information can invest in the equity market using the simple strategy of purchasing an ETF which follows the market index. A rich investor can improve his investment performance by acquiring more information. Let's think of this improvement as fund picking or stock picking. To pick up a valuable stock or a higher than average performance investment fund, investors should be able to research the corporate's financial statements, cash flow situations, the market share and the long-run aspect of the whole sector. Different language, different accounting rules and different consumer preferences make doing this research in the foreign market harder. Even if with the service from a professional or the knowledge of how to obtain such information, how to properly interprete these information and execute accordingly in the market requires even deeper understandings of
the regulatory system, the policies, the political system and the culture in a foreign country. These kinds of information are very valuable and hard to obtain. Investors may not know the differences in all these aspects which contribute to information cost asymmetry. However, we believe that they are perfectly aware of the existence of such obstacles that they will face once they put their money across borders.

4 Decompose Equity Home Bias

In order to know how much of the observed equity home bias is actually limited participation in the foreign market, we decompose equity home bias into its extensive and intensive margins.

If we know the participation rate in the stock market (both the domestic and foreign markets) \( t \) and in the foreign stock market \( t_f \), together with the average holdings of equities \( \sigma \) and of foreign equities \( \sigma_f \) conditional on participation, we can express the share of foreign equities in a country’s equity portfolio as \( \frac{t_f \sigma_f}{t \sigma} \). Let \( M_f \) denote the foreign market capitalization and \( M \) the world market capitalization. Following the literature, the degree of equity home bias is defined as one minus the ratio of the share of foreign equities in a country’s equity portfolio to the share of foreign market capitalization in the world market capitalization. It measures how far away the country’s equity portfolio is from the ideal one. The bigger the number is, the less diversified, or more home biased the portfolio is.

\[
EHB = 1 - \frac{t_f \sigma_f}{t \sigma} \frac{M_f}{M} \tag{23}
\]

If a portfolio consists of only home equities, the share of foreign equities is zero, and the degree of equity home bias is one. If the share of foreign equities in a portfolio is equal to the share of foreign market capitalization in the world market capitalization, the portfolio is perfectly diversified and has a degree of equity home bias being zero.

We can decompose EHB as follows:

\[
EHB = 1 - \frac{t_f}{t} + \left(1 - \frac{\sigma_f/\sigma}{M_f/M}\right) \frac{t_f}{t} \tag{24}
\]

\[
EHB_{ex} = 1 - \frac{t_f}{t} \tag{25}
\]

\[
EHB_{in} = \left(1 - \frac{\sigma_f/\sigma}{M_f/M}\right) \frac{t_f}{t} \tag{26}
\]

23
The extensive margin bias is defined as the proportion of equity holders who do not own any foreign equity. If the foreign equity is in all equity owners’ portfolio, then $t_t = 1$, and $EBH_{ex} = 0$. There is no bias on the extensive margin. The intensive margin measures on average how biased individual investors’ equity portfolios are conditional on positive holdings of the foreign equity. If investors on average perfectly diversify their equity portfolios, then $\frac{\sigma_f}{\sigma_M} = 1$, and $EBH_{in} = 0$. 1 $-$ $\frac{\sigma_f}{\sigma_M}$ measures how far away the equity portfolio is, at individual level, from the ideal one. Since only a fraction of equity holders own the foreign equity, $1 - \frac{\sigma_f}{\sigma_M}$ is multiplied by $\frac{t_t}{t}$ to get the measure of the intensive margin bias.

$$EBH = EBH_{ex} + EBH_{in}$$

$$1 = \frac{EBH_{ex}}{EBH} + \frac{EBH_{in}}{EBH}$$

The percentage of the observed equity home bias that should be attributed to the extensive margin is $\frac{EBH_{ex}}{EBH}$, while the rest ($1 - \frac{EBH_{ex}}{EBH}$) is attributed to the intensive margin. Now let’s look at the real equity home bias data in the U.S. in 2005 to see the exact percentage associated with each margin.

$$t_t = 0.63 \quad \frac{t_t}{t} = 0.18 \quad \frac{M_f}{M} = 0.5954$$

$EBH = 0.7 \quad EBH_{ex} = 0.37 \quad EBH_{in} = 0.33$

$$\frac{EBH_{ex}}{EBH} = 53\% \quad \frac{EBH_{in}}{EBH} = 47\%$$

In 2005\(^{14}\) the share of foreign equities in US equity portfolio is 18\%\(^{15}\). The share of foreign market capitalization in the world market capitalization is 59.54\%\(^{16}\). 63\% of individual equity investors own foreign equities directly and/or indirectly through investment funds\(^{17}\). According to the definition of equity home bias and the decomposition. The degree of equity home bias in the U.S. in 2005 is 0.7. 53\% of the bias is the extensive margin bias which has been ignored in the literature.

In the model, we assume that initial wealth follows a Pareto Distribution with shape parameter $\lambda$ and lower bound $W$. The proportion of the population with initial wealth above $W$ is $(\frac{W}{W})^\lambda$. Therefore, the proportions of the population participating in the home equity market and in the foreign equity market are $(\frac{W_h}{W})^\lambda$ and $(\frac{W_f}{W})^\lambda$ respectively. Since all

---

\(^{14}\)2005 is the most recent year for which we have the extensive margin data.

\(^{15}\)See Sercu and Vanpee \cite{sercu} and Gau et al. \cite{gau}.

\(^{16}\)Datasource: World Federation of Exchanges

\(^{17}\)Equity ownership in America, 2005 by the Investment Company Institute and the Securities Industry Association
the foreign equity investors also invest in the home equity in our model, the percentage of
equity investors who own foreign equities is \((\bar{W}_h / \bar{W}_f)^\lambda\). Therefore the extensive margin bias is
as follows:

\[
EHB_{ex} = 1 - (\bar{W}_h / \bar{W}_f)^\lambda
\]  

(29)

The bigger the gap between \(\bar{W}_h\) and \(\bar{W}_f\) is, the greater the number of persons who
hold only home equities is, the higher the extensive margin bias is. Pareto distribution is
frequently used to describe the wealth or income distribution in an economy, of which the
Gini coefficient can be expressed as a function of the shape parameter \((\frac{1}{\lambda-1})\). An economy
has a wealth distribution with a greater \(\lambda\) is more equal. According to equation \[29\], a
greater \(\lambda\) means a bigger extensive margin bias. A more equal economy exhibits a higher
degree of equity home bias because those who do purchase foreign equities own a smaller
share of the total wealth.

Analogous to an individual investor’s degree of diversification, a country’s degree of
diversification is defined as the share of foreign equities in the country’s equity portfolio:

\[
D = \frac{\int_{\bar{W}_f}^{\infty} E(D_f P_f) f(W) dW}{\int_{\bar{W}_h}^{\infty} E(D_h P_h) f(W) dW + \int_{\bar{W}_f}^{\infty} E(D_f P_f) f(W) dW}
\]

\[
= \frac{\int_{\bar{W}_f}^{\infty} \frac{W}{a} (V + \frac{Q^2}{V} + S_f^*) f(W) dW}{\int_{\bar{W}_h}^{\infty} \frac{W}{a} (V + \frac{Q^2}{V} + S_h^*) f(W) dW + \int_{\bar{W}_f}^{\infty} \frac{W}{a} (V + \frac{Q^2}{V} + S_f^*) f(W) dW}
\]

A complete home biased equity portfolio corresponds to \(D = 0\). While a fully diversifed
one shows that \(D = 0.5\). In the model, two countries are symmetric. Therefore, the foreign
market capitalization is half the world market capitalization. Following the definition of
equity home bias and the decomposition developed previously, we can express the degree of
equity home bias of the country in the model as follows:

\[
EHB = 1 - 2D
\]

\[
EHB_m = (\bar{W}_h / \bar{W}_f)^\lambda - 2D
\]
$D$ can be rewritten as:

$$D = \frac{\int_{W_f}^{\infty} \frac{W}{a} (V + \frac{Q^2}{U} + S_f^*) f(W) dW}{1 + \int_{W_f}^{\infty} \frac{W}{a} (V + \frac{Q^2}{U} + S_f^* + V + \frac{Q^2}{U} + S_h^*) f(W) dW}$$

The numerator in equation (30) can be viewed as the aggregate level of diversification among investors in the top wealth group. The more diversified these investors are, the lower the intensive margin bias is, thus the lower the degree of aggregate equity home bias is. In the denominator of equation (30), we have $\int_{W_f}^{\infty} \frac{W}{a} (V + \frac{Q^2}{U} + S_h^*) f(W) dW$, which is the aggregate demand of home equity from agents who invest in only the home market. The bigger the extensive margin bias is, the greater this term is, thus the higher the degree of aggregate equity home bias is.

5 Numerical Simulation

Mathematically, the solution to the model is a fixed point due to the nature of the rational expectations model. Asset demands, information acquisition, asset prices, and the degree of equity home bias all depend on the value of the fixed point. In the numerical simulation, we need first calculate the value of the fixed point given the value of all the other exogenous parameters\textsuperscript{18} and then compute the degree of equity home bias.

Exogenous Parameters

We assume that initial wealth follows a Pareto distribution. The lower bound is 1, the shape parameter is 2. Assumptions about the Pareto Distribution together with $a = 4$ implies that the average absolute risk aversion is 2.67\textsuperscript{19} which is considered as a reasonable value in the literature. We set the riskless rate of return to be 1.01. Following Coval [2003], we assume the mean and variance of equity payoff to be 1.2 and 4. The average supply of equity in each market is 1. The variance of equity supply is set to be 1 so that the supply volatility accounts for less than half of the price volatility.

\textsuperscript{18}The function which maps to its fixed point is an integration over another function which is continuous but has a breakpoint. The lower bound of the integration is a function which can not be explicitly expressed. Therefore, we choose the iteration approach to solve for the fixed point.

\textsuperscript{19}$\int_{\infty}^{\infty} \frac{W}{a} f(W) dW = \int_{1}^{\infty} \frac{4}{W} \frac{2}{W^2} dW = 2.67$
The convex information cost function is represented by a quadratic function. Higher information cost in the foreign market is represented by greater coefficients in both the quadratic and the linear term. The limit ($\bar{d}$) to which diversification of an individual investor converges to is set to be equal to the average share of foreign equities among the top percentile in the wealth distribution of only foreign equity holders, which is 23% according to 2010 SCF\(^{20}\). This implies that $\frac{\alpha_f}{\alpha_h} = 1/11$. The values of the four information cost parameters are chosen such that the country’s holdings of foreign equities as a share of total equity holdings is close to the data\(^{21}\). The share of foreign equities in US equity portfolio is 18% in 2005 and 23% in 2008\(^{22}\). With the parameter values given in Table 1, the share of foreign equities generated in the numerical simulation is 20%. In the model, the foreign market capitalization is half the world market capitalization. Therefore, the degree of equity home bias of the country is 0.6.

### Table 1: Parameters Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower bound of initial wealth</td>
<td>$\bar{W}$</td>
</tr>
<tr>
<td>Shape parameter of Pareto dist.</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$a$</td>
</tr>
<tr>
<td>Mean of equity payoff</td>
<td>$\bar{R}$</td>
</tr>
<tr>
<td>Variance of equity payoff</td>
<td>$1/V$</td>
</tr>
<tr>
<td>Mean of equity supply</td>
<td>$\bar{Z}$</td>
</tr>
<tr>
<td>Variance of equity supply</td>
<td>$U$</td>
</tr>
<tr>
<td>Riskless rate of return</td>
<td>$r$</td>
</tr>
<tr>
<td>Information cost in the home market</td>
<td>$\alpha_h S^2 + \beta_h S$</td>
</tr>
<tr>
<td>Information cost in the foreign market</td>
<td>$\alpha_f S^2 + \beta_f S$</td>
</tr>
</tbody>
</table>

**Extensive Margin**

91.1 million Americans, 40% of US civilian noninstitutional population, own equities in 2005\(^{23}\). 63% of these individual equity investors own foreign equities directly and/or indirectly. The model predicts that agents with initial wealth above $\bar{W}_h$ own equities, agents with initial wealth above $\bar{W}_f$ own foreign equities. Given our assumptions about the Pareto distribution, it implies that $(\frac{1}{\bar{W}_h})^2 = 0.4$ and $(\frac{\bar{W}_h}{\bar{W}_f})^2 = 0.63$. Thus $\bar{W}_h = 1.58$ and $\bar{W}_f = 1.99$.

\(^{20}\)This number is shown in Figure 4b.

\(^{21}\)We searched over a domain for values of these parameters which generate the degree of equity home bias closest to the data. The domain is chosen based on my understandings of the mechanism and through trial and error.

\(^{22}\)See Coeurdacier and Rey [2011].

\(^{23}\)50% US households own equities.
Fixed Point

In the discussion of the model solution, we treat $\bar{\rho}$ and $Q$ as exogenous parameters. However, they are actually endogenously determined in the model. $\bar{\rho}$ is the average risk tolerance among all participants in a market. $Q$ is the weighted average precision level of private information among all participants in a market. Participants in each market are either local or foreigners. The model predicts that local participants have initial wealth above $\bar{W}_h$, and foreign participants have initial wealth above $\bar{W}_f$. Therefore we can express $\bar{\rho}$ and $Q$ as follows:

$$\bar{\rho} = \int_{\bar{W}_h}^{\infty} \frac{W}{a} f(W) dW + \int_{\bar{W}_f}^{\infty} \frac{W}{a} f(W) dW$$  \hspace{1cm} (31)$$

$$Q = \int_{\bar{W}_h}^{\infty} \frac{W}{a} S^*_h(W) f(W) dW + \int_{\bar{W}_f}^{\infty} \frac{W}{a} S^*_f(W) f(W) dW$$  \hspace{1cm} (32)$$

Clearly, the values of $\bar{\rho}$ and $Q$ are determined by the wealth thresholds of participation in both markets as well as the amount of information that each participant purchases. However, we can see in the discussion of solution that the wealth thresholds and optimal information acquisition are determined by the value of $\bar{\rho}$ and $Q$. Agents do cost-benefit analysis when they decide whether to participate in the market and how much information to purchase. The benefit is affected by who else are participating ($\bar{\rho}$) and how informative other participants are ($Q$). Therefore, the solution to the model is a fixed point. We prove in appendix (D) that the existence of the fixed point, or the solution. In the numerical exercise, we need first calculate the fixed point given the values of exogenous parameters, then calculate demands of equities, information acquisition etc.. Each time we change the value of any exogenous parameter, we need recalculate the fixed point.

If the function which maps to the fixed point is a contraction mapping, we can easily implement the iteration method to solve for the fixed point. However, the mapping function turns out to be not a contraction mapping. We illustrate the diverging nature of the mapping function using a simple one dimension function $g(x)$ in Figure 2a. Figure 2b shows the modifications that we made to a simple iteration approach which allow us to reach the fixed point of an oscillating diverging mapping. We start from one point ($x_1$) in the neighborhood of the fixed point, and do two iterations each time. If the size of the second step ($|x_3 - x_2|$) is smaller than of the first step ($|x_2 - x_1|$), the process continues on. If on the contrary the size of the second step is bigger than of the first step, we rechoose the starting point as the middle place ($x'_1$) of the first two points, and restart the process. We stop the process when
Figure 2: Fixed Point

(a) Oscillating Divergence

(b) Iteration Approach

the size of one step is small enough (which means we are close enough to the fixed point.\textsuperscript{24}

Results

With parameter values given in Table 1, the share of a country’s aggregate equity investment in the foreign market is 20%. 40% of the whole population buy equities and 63% of equity investors own the foreign equity. It can match pretty closely to the aggregate home bias as well as its extensive margin.

Figure 3 summarize the main results from the numerical exercise. Consistent with the literature, share of wealth invested in equity and information acquisition increase in wealth. Figure 3a plots the mean of wealth share in equity against wealth. Agents with wealth between 1.58 and 1.99 invest in only domestic equity. For those who invest in both market, share of foreign equity is always lower than share of domestic equity. Figure 3b plots the mean of information purchased against wealth. Information purchased in the foreign market is always lower than in the domestic market. Agents with wealth between 1.58 and 3.44 purchase only domestic information.

Figure 3c plots degree of diversification (share of foreign equity in equity portfolio) against wealth. The horizontal line in Figure 3c is $\bar{d}$, towards which diversification is converging. Although share of wealth invested in both home equity and foreign equity increase in wealth, we can see that agents tend to sell both domestic and foreign equities to lower their exposure.

\textsuperscript{24}Matlab codes of the simulation exercise is available at my website: http://www.yyliu.weebly.com
Figure 3: Numerical Example

(a) Share of Wealth invested in equity  (b) Amount of information purchased  (c) Share of total equity investment in foreign equity

share of total equity investment in foreign equity is decreasing. The kink in the plot happens where the wealth threshold of foreign information acquisition is. When agents start buying foreign information, diversification decreases slower. This negative relationship between international diversification and wealth is later documented in the empirical evidence section. Figure 4b is a counterpart plot using real data.

In the household finance literature, it is documented that the share of risky assets increases in wealth. On average, Rich people does a better job managing and investing his wealth. We may also expect that rich ones are more diversified international. On the contrary, diversification decreases in wealth as shown in the numerical example and later proven in the empirical evidence section. This fact helps explain equity home bias. Agents not only always invest a larger share of wealth in home equity than in foreign equity, but also increases the share of home equity much faster than the share of foreign equity as wealth goes up. Wealthy people have substantial influences on asset demand. Their diversification level could have a great effect on country’s aggregate diversification level. However, they are all biased towards home and are becoming even more biased as their wealth increases. This fact is very conveniently consistent with the observed high degree of aggregate equity home bias.

6 Empirical Evidence

In the theory part, we see that the relationship between international diversification and wealth could be monotonic positive, monotonic negative or non-monotonic. In this section, we document empirical evidences of a monotonic negative relationship. We also provide empirically supports of the prediction that information acquisition increases in wealth.
Table 2: Summary Statistics

<table>
<thead>
<tr>
<th>Sample Mean</th>
<th>Direct Foreign Stock Holders</th>
<th>Direct Stock Holders</th>
<th>Whole Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Wealth</td>
<td>1,663,889</td>
<td>885,564</td>
<td>223,446</td>
</tr>
<tr>
<td>Household’s Income</td>
<td>238,334</td>
<td>170,413</td>
<td>77,060</td>
</tr>
<tr>
<td>Stocks</td>
<td>565,095</td>
<td>203,339</td>
<td>30,682</td>
</tr>
<tr>
<td>Foreign Stocks</td>
<td>147,246</td>
<td>21,144</td>
<td>3,191</td>
</tr>
<tr>
<td>Share of Foreign Stocks</td>
<td>33.06%</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Age</td>
<td>55</td>
<td>54</td>
<td>51</td>
</tr>
<tr>
<td>Married</td>
<td>0.69</td>
<td>0.72</td>
<td>0.58</td>
</tr>
<tr>
<td>White</td>
<td>0.86</td>
<td>0.86</td>
<td>0.71</td>
</tr>
<tr>
<td>College</td>
<td>0.42</td>
<td>0.38</td>
<td>0.25</td>
</tr>
<tr>
<td>Advanced Degree</td>
<td>0.40</td>
<td>0.30</td>
<td>0.13</td>
</tr>
<tr>
<td>Shop Around</td>
<td>3.63</td>
<td>3.51</td>
<td>3.06</td>
</tr>
</tbody>
</table>

Note: Shop around is a self-reported degree of information shopping on a scale from 1 to 5. Married, White, College, Advanced Degree are all dummy variables. All data are weighted. SCF implements a multiple imputation procedure to correct for missing data. Therefore all estimates are obtained by combining results across multiple implicates.

Our analysis is based on 2010 Survey of Consumer Finance. SCF is a cross-section survey of US families balance sheets, pensions, income, and demographic characteristics. It covers a representative sample of the entire population and oversamples the wealthy who have a disproportionate influence of asset demands. The survey breaks households’ holdings of stocks into direct holdings through a brokerage account and indirect holdings through investment funds. It distinguishes direct foreign equity holdings from direct domestic equity holdings. However, there is lack of information about indirect holdings of foreign stocks. We thus restrict our analysis to the sub-sample of direct foreign stock holders. Households in the sub-sample account for 2.2% of the whole sample. However, they own 16% of the total financial wealth and 40% of the total direct equity holdings of the whole sample. The level of international diversification is measured as the share of direct foreign stock holdings, i.e. the percentage of total direct stock holdings invested in foreign stocks.

In the literature of household finance, households’ portfolio share is defined either as the share of net worth invested in risky assets, or the share of financial wealth invested in public stocks. Following the narrower definition of the portofolio share in the literature, wealth in our analysis is financial wealth. It includes transaction accounts, certificate of deposits(CDs), bonds, saving bonds, stocks, pooled investment funds, retirement accounts, cash value life insurance and other managed assets. We use the same definition of financial wealth as the one used in 2010 SCF chartbook. See Bricker et al. for the definition of this variable and its subcategories.
similar results using a narrower definition of financial wealth, which includes only transaction accounts, CDs, bonds, saving bonds, and stocks. Regression results using both definitions are reported in Table 3.

According to 2010 SCF, 15.1% of US households directly own stocks and 2.2% directly own foreign stocks. Table 2 compare the means of a list of variables across three samples: the whole sample, the subsample of direct stock owners, and the subsample of direct foreign stock owners. Figure 4a plots share of households directly own equities against wealth percentiles. We can see from both Table 2 and Figure 4a that direct foreign equity ownership is concentrated among the wealthy. Those who directly own foreign equities are substantially wealthier, has higher income, shop for more information, and are much better educated than average. The rank of wealth and information shopping among three groups of investors, represented by three columns in Table 2 is consistent with model predictions. Figure 4b plots the share of foreign equity in direct equity investments against wealth percentiles using the subsample of only foreign equity owners. Similar to Figure 3c in the numerical exercise section, figure 4b suggests that diversification decreases in wealth.

Figure 4: Wealth Percentile Plots

The model predicts that wealthy purchase more information thus invest a larger share of wealth in equities. However, depending the relative size of the information cost at home and abroad, level of international diversification could be increasing, decreasing or first decreasing then increasing in wealth. We would like to know whether there exists a clear pattern between international diversification and wealth as predicted by the model, and which one of these three is true. To answer these questions, we estimate simple OLS regressions of diversification on wealth. Results from four model specifications are reported. The dependent variable is logarithm of international diversification. In Model (1) and (3), our interested explanatory

\footnote{Similar tables and figures based on previous years of SCF data can be found in Nechio 2010}
Table 3: Wealth Effect on International Diversification

<table>
<thead>
<tr>
<th>Dependent Variable: log(International Diversification)</th>
<th>Model (1)</th>
<th>Model (2)</th>
<th>Model (3)</th>
<th>Model (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Wealth)</td>
<td>-0.347***</td>
<td>-0.555</td>
<td>-0.268***</td>
<td>-0.818*</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.535)</td>
<td>(0.053)</td>
<td>(0.449)</td>
</tr>
<tr>
<td>log(Wealth)^2</td>
<td>0.008</td>
<td>0.023</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.023)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>33.96%</td>
<td>34.03%</td>
<td>31.96%</td>
<td>32.51%</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>29.08%</td>
<td>28.93%</td>
<td>26.93%</td>
<td>27.28%</td>
</tr>
</tbody>
</table>

***Significant at 1% level. **Significant at 5% level. *Significant at 10% level.
Heteroskedasticity-robust standard errors are reported in parenthesis. All data are weighted.
SCF implements a multiple imputation procedure to correct for missing data.
Therefore all estimates are obtained by combining results across multiple implicates.
(1) (2): Wealth is Financial Wealth.
(3) (4): Wealth is defined using the narrower definition of Financial Wealth.
In all model specifications, we controlled for demographic characteristics and information sources. See appendix E for a complete list of controlled variables.

The variable is logarithm of wealth. In model (2) and (4), we add a quadratic term. Wealth is financial wealth. Model (3) and (4) use the narrower definition of financial wealth. Coefficient estimates on logarithm of wealth is negative and significant if quadratic term is not included. The results suggest that a 1% increase in wealth gives rise to a 0.3% percent decrease in diversification.

The model predicts that information purchasing increases in wealth. Figure 4c plots mean value of self reported degree of information shopping against wealth percentiles. The solid line is obtained using the whole sample while the dashed line using subsample of direct equity owners. Consistent with the prediction, both line are upward sloping and dashed line is above the solid line.

Table 4 reports estimates of wealth effect on information acquisition using ordinal logistic models. In model (1), we use self reported degree of shopping as the measure of information acquisition. It is a categorical variable with ordered five values. In model (2), we tranform five ordered categories to three: almost no shopping; moderate shopping; and a great deal of shopping. In model (3), information acquisition is measured as frequency of trading. It has four ordered values: no trading, no more than one trade per month; more than one trade per month but no more than one trade per week; and more than one trade per week. We think that more information that an investor has, more transactions will be made. Each

---

27 In the show card presented to respondents in the survey, information shopping are scaled from 1 to 5. Below 1 is printed "Almost no shopping"; below 3 is printed "Moderate shopping"; and below 5 is printed "A great deal of shopping". Following the definition of summary variables used in SCF Chartbook, we define 1 as "no shopping", 2 to 4 as "moderate shopping" and 5 as "a great deal of shopping".

33
### Table 4: Wealth Effect on Information Acquisition

<table>
<thead>
<tr>
<th>Dependent Var.</th>
<th>Model (1)</th>
<th>Model (2)</th>
<th>Model (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Information Shopping</td>
<td>Information Shopping</td>
<td>Frequency of Trading</td>
</tr>
<tr>
<td>log(Wealth)</td>
<td>0.038***</td>
<td>0.031***</td>
<td>0.501***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.01)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>Equity Owner</td>
<td>0.269***</td>
<td>0.274***</td>
<td>1.752***</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.118)</td>
</tr>
<tr>
<td>Foreign Equity Owner</td>
<td>0.051</td>
<td>0.153</td>
<td>0.646***</td>
</tr>
<tr>
<td></td>
<td>(0.163)</td>
<td>(0.17)</td>
<td>(0.213)</td>
</tr>
<tr>
<td>Pseudo R-squared</td>
<td>7.51%</td>
<td>9.58%</td>
<td>34.97%</td>
</tr>
</tbody>
</table>

***Significant at 1% level. **Significant at 5% level. *Significant at 10% level.

Heteroskedasticity-robust standard errors are reported in parenthesis. All data are weighted. SCF implements a multiple imputation procedure to correct for missing data. Therefore all estimates are obtained by combining results across multiple implicates. Wealth is Financial Wealth.

In all model specifications, we controlled for demographic characteristics, working status and information sources. See appendix E for a complete list of controlled variables.

Time an investor acquire some information which indicating a profitable opportunity, he will trade accordingly to exploit the opportunity. Regression results are consistent with model predictions. According to estimates of model (1) and (2), for a 1% increase in wealth, we would expect a 0.03 to 0.04 increase in log odds of being in a higher level of information shopping. Owning equities direct is predicted to increase log odds of being in a higher level of information shopping by 0.3. According to estimates of model (3), for a 1% increase in wealth, we would expect a 0.5 increase in log odds of trading at a higher level of frequency. Owning equities directly is predicted to increase log odds of trading at a higher level of frequency by 1.75. Owning foreign equities directly is predicted to increase log odds of trading at a higher level of frequency by 0.65.

### 7 Conclusion

Based on the understandings of several well documented stylized facts, we believe that the lackness of international diversification in a country’s equity portfolio can be explained, at least partly, by entry cost asymmetry and information cost asymmetry between the home and foreign equity markets. To convey that belief, we incorporate entry costs, endogenous information acquisition and wealth heterogeneity into a rational noisy expectations model in the context of two symmetric countries. The numerical simulation can get very close to the degree of aggregate equity home bias in the U.S. because the model generates heterogenous portfolio demands among investors. According to the model’s predictions, investors can be
categorized into three groups in terms of equity ownership. Top wealth group owns both home and foreign equities, bottom wealth group does not own any equity, and the group in between owns only home equities. The group in between holds equity portfolios completely biased toward home. Their contributions to the aggregate equity home bias are defined as the extensive margin bias. The intensive margin bias states that investors hold more of home equities than of foreign equities conditional on positive holdings of both. Within the top wealth group, home bias could be increasing or decreasing in wealth depending on the relative increasing speed of the marginal information cost home vs foreign. Our finding of an increasing degree of equity home bias in wealth endows the theory more power toward explaining the high degree of aggregate equity home bias. However, due to data limitations, we can not exactly measure the share of foreign equities in a household’s equity portfolio. Rather, in our empirical analysis, we use the share of foreign equities in a household’s equity portfolio which is directly owned. Further study using dataset which includes detailed information about international diversification of households’ indirect equity holdings is required to discover further the relationship between wealth and international diversification.

Appendices

A Optimal Portfolio

An agent’s optimization problem at period three is:

$$\max \mathbb{E}(-exp(-\frac{W}{\rho}))$$

$$W = W_0r + \sum_{i=h,f} \{D_i(R_i - rP_i) - c_i(S_i) - F_i\}$$

The agent’s posterior belief about the equity payoff is: $$\tilde{R}_i \sim N(E_i, V_i^{-1})$$. Therefore final wealth follows a normal distribution:

$$W \sim N(W_0r + \sum_{i=h,f} \{D_i(E_i - rP_i) - c_i(S_i) - F_i\}, \sum_{i=h,f} D_i^2V_i^{-1})$$
According to the moment generating function of the normal distribution, we get:

$$\mathbb{E}(-\exp(-\frac{W}{\rho})) = -\exp\left(-\frac{W_0r}{\rho} - \sum_{i=h,f} \left(\frac{D_i(E_i - rP_i) - c_i(S_i) - F_i - D_iV_i^{-1}}{\rho} \frac{D_iV_i^{-1}}{2\rho^2}\right)\right)$$  (33)

The first order condition tells us that the optimal portfolio demand is:

$$D_i = \rho V_i (E_i - rP_i)$$  (34)

**B Optimal Amount of Information**

The highest expected utility achieved with optimal portfolio is:

$$v = -\exp(-\frac{W_0r}{\rho}) \prod_{i=h,f} \left(\exp\left(\frac{F_i + c_i(S_i)}{\rho}\right) \exp\left(-\frac{(E_i - rP_i)^2V_i}{2}\right)\right)$$  (35)

Expectation of $v$ based on the joint distribution of $\tilde{Y}_i$ and $\tilde{P}_i$ is:

$$\mathbb{E}_{\tilde{Y}_i, \tilde{P}_i}(v) = -\exp(-\frac{W_0r}{\rho}) \prod_{i=h,f} \left(\exp\left(\frac{F_i + c_i(S_i)}{\rho}\right) \mathbb{E}_{\tilde{Y}_i, \tilde{P}_i} \exp\left(-\frac{(E_i - rP_i)^2V_i}{2}\right)\right)$$  (36)

We need to know:

$$\mathbb{E}_{\tilde{Y}_i, \tilde{P}_i} \exp\left(-\frac{(E_i - rP_i)^2V_i}{2}\right)$$

Let $J = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} = \begin{bmatrix} \tilde{Y}_i - R \\ \tilde{P}_i - (A_0 + A_1\tilde{R} - A_2\tilde{Z}) \end{bmatrix}$, $\Sigma = \begin{bmatrix} V^{-1} + S_i^{-1} & A_iV^{-1} \\ A_iV^{-1} & A_i^2V^{-1} + A_2^2U \end{bmatrix}$.

Then $J \sim N(0, \Sigma)$. The density function is:

$$f(J) = \frac{1}{(2\pi)^{\sqrt{|\Sigma|}}} \exp\left(-\frac{1}{2}J^T\Sigma^{-1}J\right) = \frac{1}{(2\pi)^{\sqrt{|\Sigma|}}} \exp\left(-\frac{1}{2}[J_1^2\beta_{11} + 2J_1J_2\beta_{12} + J_2^2\beta_{22}]\right)$$

Where

$$\Sigma^{-1} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$$
And

\[
|\Sigma| = \frac{A_1^2}{Q^2 U} S_i \left(\frac{Q^2}{U} + V + S_i\right)
\]

\[
\beta_{12} = \frac{-\frac{Q^2}{U} S_i}{A_1 \left(\frac{Q^2}{U} + V + S_i\right)}
\]

\[
\beta_{11} = \frac{(\frac{Q^2}{U} + V) S_i}{\frac{Q^2}{U} + V + S_i}
\]

\[
\beta_{22} = \frac{(V + S_i) \frac{Q^2}{U}}{A_1 \left(\frac{Q^2}{U} + V + S_i\right)}
\]

And

\[
E_i - rP_i = B_{0i} + B_{1i} J_1 + (B_{2i} - R) J_2
\]

Where

\[
B_{0i} = \bar{R} - r(A_0 + A_1 \bar{R} - A_2 \bar{Z})
\]

\[
B_{1i} = \frac{S_i}{\frac{Q^2}{U} + V + S_i}
\]

\[
B_{2i} = \frac{\frac{Q^2}{U}}{A_1 \left(\frac{Q^2}{U} + V + S_i\right)}
\]

And

\[
(E_i - rP_i)^2 V_i = \alpha_{00} + 2\alpha_{01} J_1 + 2\alpha_{02} J_2 + 2\alpha_{12} J_1 J_2 + \alpha_{11} J_1^2 + \alpha_{22} J_2^2
\]

Where

\[
\alpha_{00} = B_{0i}^2 V_i
\]

\[
\alpha_{01} = B_{0i} B_{1i} V_i
\]

\[
\alpha_{02} = B_{0i} (B_{2i} - r) V_i
\]

\[
\alpha_{11} = B_{1i}^2 V_i
\]

\[
\alpha_{12} = B_{1i} (B_{2i} - r) V_i
\]

\[
\alpha_{22} = (B_{2i} - r)^2 V_i
\]
\[E_{\tilde{Y}, \tilde{P}} \exp(-\frac{(E_i - rP)^2 V_i}{2})\]

\[= \int_{J_1} \int_{J_2} \exp(-\frac{(E_i - rP)^2 V_i}{2}) f(J) dJ_1 dJ_2\]

\[= \int_{J_1} \int_{J_2} \frac{1}{(2\pi) \sqrt{|\Sigma|}} \exp \left(-0.5 \left(\frac{\alpha_{00} + 2\alpha_{01} J_1 + 2\alpha_{02} J_2 + 2(\alpha_{12} + \beta_{12}) J_1 J_2 + (\alpha_{11} + \beta_{11}) J_1^2 + (\alpha_{22} + \beta_{22}) J_2^2}{\alpha_{11} + \beta_{11}}\right)\right) dJ_1 dJ_2\]

\[= \frac{1}{(2\pi) \sqrt{|\Sigma|}} \int_{J_2} \exp \left(-0.5 \left(\frac{\alpha_{00} + (\alpha_{22} + \beta_{22}) J_2^2 + 2\alpha_{02} J_2}{\alpha_{11} + \beta_{11}}\right)\right) dJ_2\]

\[= \frac{1}{\sqrt{(2\pi)|\Sigma|(\alpha_{11} + \beta_{11})}} \int_{J_2} \exp \left(-0.5 \left(\frac{\alpha_{00}}{\alpha_{11} + \beta_{11}} + \frac{\alpha_{01}^2}{\alpha_{11} + \beta_{11}} + \frac{\alpha_{02}}{\alpha_{11} + \beta_{11}} - \frac{(\alpha_{12} + \beta_{12})^2}{\alpha_{11} + \beta_{11}}\right)\right) dJ_2\]

\[\exp \left(-0.5 \left(\frac{\alpha_{00} - \alpha_{01}^2}{\alpha_{11} + \beta_{11}} - \frac{(\alpha_{02} - \alpha_{01}(\alpha_{12} + \beta_{12}))^2}{\alpha_{22} + \beta_{22} - \frac{(\alpha_{12} + \beta_{12})^2}{\alpha_{11} + \beta_{11}}}\right)\right)\]

\[= \sqrt{|\Sigma|(\alpha_{11} + \beta_{11})(\alpha_{22} + \beta_{22} - \frac{(\alpha_{12} + \beta_{12})^2}{\alpha_{11} + \beta_{11}})}\]

Since

\[\alpha_{11} + \beta_{11} = S_i\]

\[\alpha_{12} + \beta_{12} = -S_i r\]

\[\alpha_{22} + \beta_{22} = \frac{Q^2 (1 - A_i r)^2}{A_i^2} + r^2 (V + S_i)\]

So

\[\alpha_{22} + \beta_{22} - \frac{(\alpha_{12} + \beta_{12})^2}{\alpha_{11} + \beta_{11}} = \frac{Q^2 (1 - A_i r)^2}{A_i^2} + r^2 V\]

38
Thus
\[ |\Sigma|(\alpha_{11} + \beta_{11})(\alpha_{22} + \beta_{22} - \frac{\alpha_{12} + \beta_{12})^2}{\alpha_{11} + \beta_{11}}) = \frac{B}{V + S_i + \frac{Q^2}{I}} \]
\[ B = \left((1 - A_1r)^2 \frac{1}{V} + (A_1r)^2 \frac{U}{Q^2}\right)^{-1} \]

Let
\[ \alpha_{00} = \frac{\alpha_{01}^2}{\alpha_{11} + \beta_{11}} - \frac{(\alpha_{02} - \alpha_{01}(\alpha_{12} + \beta_{12}))^2}{\alpha_{22} + \beta_{22} - \frac{(\alpha_{12} + \beta_{12})^2}{\alpha_{11} + \beta_{11}}} = A \]

Then
\[ A = B^2_0 B \]
\[ = \left(\frac{\bar{Z}}{\bar{\rho}V + \bar{\rho}\frac{Q^2}{I} + Q}\right)^2 B \]

Therefore
\[ E_{\tilde{Y}, \tilde{P}}(V) = -\exp\left(-\frac{W_0r}{\rho}\prod_{i=h,f} \exp\left(\frac{c_i(S_i) + F_i}{\rho} - \frac{1}{2} A\right) \sqrt{\frac{B}{V + S_i + \frac{Q^2}{I}}} \right) \]  (37)

Where
\[ A = \left(\frac{\bar{Z}}{\bar{\rho}V + \bar{\rho}\frac{Q^2}{I} + Q}\right)^2 B \]
\[ B = \left((1 - A_1r)^2 \frac{1}{V} + (A_1r)^2 \frac{U}{Q^2}\right)^{-1} \]

At period two, agents choose \( S_i^* \) (\( S_i^* \) must be non-negative) to maximize equation (37). Take first order derivative of equation (37) with respect to \( S_i \):
\[ c'_i(S_i) = \frac{\rho}{V + S_i + \frac{Q^2}{I}} \]  (38)

Proposition 2 follows from equation (38).
C Share of Equity in Wealth

\[ E(D_i P_i) = E(D_i) E(P_i) + \text{COV}(D_i, P_i) \]

And

\[ E(P_i) = \frac{\bar{R}}{r} - \frac{\bar{Z}}{r(\bar{\rho}V + \bar{\rho}Q^2 + Q)} \]  \hspace{1cm} (39)\]

\[ E(D_i) = \rho V_i (\bar{R} - r E(P_i)) = \rho V_i \frac{\bar{Z}}{r(\bar{\rho}V + \bar{\rho}Q^2 + Q)} \]  \hspace{1cm} (40)\]

\[ E(D_i) E(P_i) = \rho (V + S_i + \frac{Q^2}{U}) \frac{\bar{Z} \bar{R}(\bar{\rho}V + \bar{\rho}Q^2 + Q) - \bar{Z}^2}{r(\bar{\rho}V + \bar{\rho}Q^2 + Q)^2} \]  \hspace{1cm} (41)\]

Mean demand of equity \( E(D_i) \) is positive if mean supply of equity \( \bar{Z} \) is positive. Mean price \( E(P_i) \) is positive if mean supply of equity is much lower than mean payoff. And

\[ \text{COV}(D_i, P_i) = \rho V_i \left( \text{COV}(E_i, P_i) - r \text{VAR}(P_i) \right) \]

\[ = \rho V_i \left( \frac{A_1}{V} - r \text{VAR}(P_i) \right) \]

\[ = \rho (V + S_i + \frac{Q^2}{U}) \frac{-U - \bar{\rho}Q}{r(\bar{\rho}V + \bar{\rho}Q^2 + Q)^2} \]

Covariance between demand and price is always negative. Demand of equity is linear in excess return \( E_i - R P_i \). Excess return positively depends on posterior mean payoff, negatively depends on price. Therefore the covariance between demand and price positively depends on the covariance between posterior mean payoff and price, negatively depends on the variance of price. When the variance of price is much bigger than the covariance, we have a very big negative covariance between demand and price. Price is linear in payoff and supply. Therefore, variance of price is a weighted sum of payoff variance and supply variance. Agents’ posterior belief about equity payoff and market price both contain information conveyed by prior belief of equity payoff. Therefore covariance between posterior mean and price is just part of prior payoff variance. So when supply variance is big, we have a big gap between variance of price and covariance between price and posterior mean, thus a big negative covariance between demand and price.

\[ E(D_i P_i) = \rho (V + S_i + \frac{Q^2}{U}) C \]  \hspace{1cm} (42)
\[ C = \frac{\bar{Z}\bar{R}(\bar{\rho}V + \bar{\rho}Q^2 + Q) - \bar{Z}^2 - \bar{\rho}Q - U}{r(\bar{\rho}V + \bar{\rho}Q^2 + Q)^2} \]

Suppose the mean payoff is much bigger than mean supply, then price and demand both have positive mean. When negative covariance of demand and price overpower the product of a positive demand and positive price, mean value of investment in equity is negative. It means that the negative covariance is so strong that demand and price have different signs most of the time.

D Existence of the solution

Assume information cost function is: 
\[ c_i(S_i) = \alpha_i S_i^2 + \beta_i S_i \]
where both \(\alpha\) and \(\beta\) are positive for \(i = h, f\). Assume also that \(\alpha_f > \alpha_h\) and \(\beta_f > \beta_h\). Assume that the shape parameter of pareto distribution is 2 and the lower bound of initial wealth is 1. Thus the pdf function of wealth distribution is 
\[ f(W) = \frac{2}{W^3}. \]

Define a compact set 
\[ H = H_1 \times H_2 \text{ in } \mathbb{R}^2 \text{ where } H_1 = [0, \frac{4}{a}] \text{ and } H_2 = [0, \frac{4}{a\sqrt{a\alpha_h}}]. \]

Let \(x_1 \in B_1\), \(x_2 \in B_2\), so 
\[ x = [x_1, x_2]' \in B. \]

Define the mapping 
\[ G(x) : B \to \mathbb{R}^2 \]
as follows:

\[ G(x) = [G_1(x), G_2(x)]' \]

\[ G_1(x) = \int_{W_h}^{\infty} W f(W)dW + \int_{W_f}^{\infty} W f(W)dW \]  \hspace{1cm} (44)

\[ G_2(x) = \int_{W_h}^{\infty} W S_i^* f(W)dW + \int_{W_f}^{\infty} W S_i^* f(W)dW \]  \hspace{1cm} (45)

Where 
\[ S_i^* = \max \{0, \tilde{S}_i\} \text{ for } i = h, f, \] , and \(\tilde{S}_i\) is given by:

\[ 2\alpha_i \tilde{S}_i + \beta_i = \frac{W}{2a(V + \frac{x^2}{U} + \bar{S}_i)} \]

\(\bar{W}_i\) is given by:

\[ F_i + \alpha_i S_i^* + \beta_i S_i^* = \frac{W}{2a} (\ln \frac{V + \frac{x^2}{U}}{B} + A) \]
Where

\[ A = \frac{Z^2}{x_1^2 V + x_1^2 \frac{x_2^2}{U} + U + 2x_1x_2} \]
\[ B = \frac{x_1 V + x_1 \frac{x_2^2}{U} + x_2}{x_1^2 V + x_1^2 \frac{x_2^2}{U} + U + 2x_1x_2} \]

We want employ Brouwer's Theorum to determine that there exists a \( x^* \in B \) such that \( G(x^*) = x^* \). Therefore, we need to prove two things: \( G \) is continuous in \( x \) and \( G \) maps into itself.

For \( i = h, f \), \( S_i^* \) is continuous in \( x_2 \), \( W_i \) is continuous in \( x_1 \) and \( x_2 \), thus \( G_1(x_1, x_2) \) and \( G_2(x_1, x_2) \) are continuous in \( x = [x_1, x_2]' \). So \( G(x) \) is continuous.

We have

\[ 0 < G_1(x) < \frac{2}{a} \mathbb{E}(W) = \frac{4}{a} \]

Since

\[ S_i^{*2} < (S_i^* + \frac{\beta_i}{2 \alpha_i})(S_i^* + V + \frac{x_2^2}{U}) = \frac{W}{2a \alpha_i} < \frac{W}{2a \alpha_h} \]

\[ S_i^* < \sqrt{\frac{W}{2a \alpha_h}} \]

We have

\[ 0 < G_2(x) < \frac{2}{a \sqrt{a \alpha_h}} \int_1^\infty \frac{1}{W^{1.5}} dW = \frac{4}{a \sqrt{a \alpha_h}} \]

Therefore \( G(x) \) maps into itself, which establishes the results.

E Controlled Variables

Controlled variables in regressions reported in Table 3: age, marrital status, cohort: age below 35; age between 45 and 54; age between 55 and 64; age between 65 and 74; age above 75, education: no highschool; high school graudates; some college; post-college advanced degree, ethnicity: black; hispanic; others, information sources: research by self; use a financial planner; use a financial professional; use information from friends, relatives, business contacts, investment club; Use online services or information from internet; read magazines, newspapers. use information from TV, radio, advertisement; Call around.

In Table 4 we controlled for all variables that are controlled in Table 3 and add workstatus: self-employed/partnership; reitred/disabled and students/homemaker/misc. not work-
ing age 65 or older; other groups not working.

References


Coval, J. D. (2003). International capital flows when investors have local information. (04-026).


46